on the segment $E = [1/(2\Lambda\sqrt{\Lambda}+1), 1/(2\sqrt{\Lambda})]$, where $\Lambda \geq 16$. Then

$$\min_{X \in E} f(X) \geq \frac{\Lambda\sqrt{\Lambda}}{5}.$$

*Proof:* Since $f''(X) \leq 0$ on the segment $E$, then the function $f(X)$ is a concave one and it reaches its minimum value on the borders of $E$

$$f\left(\frac{1}{2\Lambda\sqrt{\Lambda}+1}\right) = \frac{\Lambda^3/(4\Lambda\sqrt{\Lambda}+2)}{1+\dfrac{1+\Lambda^2}{(2\Lambda\sqrt{\Lambda}+1)^2}} \geq \frac{\Lambda\sqrt{\Lambda}}{5}$$

$$f\left(\frac{1}{2\sqrt{\Lambda}}\right) = \frac{(1/4)\Lambda^2\sqrt{\Lambda}}{1+1/(4\Lambda)+\Lambda^2/4} \geq \frac{\Lambda\sqrt{\Lambda}}{5}. \qquad \square$$

*Lemma 5:* The assertion of the theorem holds for a 2-dimensional case.

*Proof:* Assume the same initial conditions $\varphi_0$ and $\delta_0$ as in Lemma 2 with $\epsilon$ sufficiently small. Then, Lemma 2 implies that $w_6 \in W(\Gamma, \Lambda)$ for some $\Gamma \simeq \epsilon$ and $\Lambda \simeq \lambda$. Note that if $w_k \in W(\Gamma, \Lambda)$ then from condition i) we obtain the following bounds for $\varphi_k^1/\varphi_{k-1}^1 = \varphi_k^1/\varphi_k^2$:

$$-\Lambda - \frac{1}{2\sqrt{\Lambda}} \leq \frac{\varphi_k^1}{\varphi_{k-1}^1} \leq -\Lambda + \frac{1}{2\sqrt{\Lambda}}.$$

Therefore

$$\left|\frac{\varphi_k^1}{\varphi_{k-1}^1}\right| \geq \frac{2\Lambda\sqrt{\Lambda}-1}{2\sqrt{\Lambda}} \tag{16}$$

and

$$\left|\frac{\varphi_k^1}{\varphi_{k-1}^1}\right| \leq \frac{2\Lambda\sqrt{\Lambda}+1}{2\sqrt{\Lambda}}. \tag{17}$$

Lemma 1 states that if $w_k \in W(\Gamma, \Lambda)$ then the only way for $w_{k+1}$ to leave the set $W(\Gamma, \Lambda)$ is to violate condition i). From (16) we derive that $(2\Lambda\sqrt{\Lambda}-1)/(2\sqrt{\Lambda}) > 15$ for the considered values of $\Lambda$. Thus, we conclude that for any $\epsilon$ there will be a time $T$ such that $w_T \notin W(\Gamma, \Lambda)$. On the other hand, the estimate given in (17) implies that $T \to \infty$ as $\epsilon \to 0$. This estimate implies that the conditions of Lemma 3 hold for $\varphi_{T-1}$. Applying subsequently Lemmas 3 and 4 we obtain the proposition of the lemma. $\square$

We have proved the theorem for the 2-dimensional case. Other dimensions may be considered in the same way. Indeed, if we choose

$$\varphi_0^3 = \cdots = \varphi_0^N = 0, \qquad \delta_0^3 = \cdots = \delta_0^N = 0$$

then all vectors $\varphi_t$ and $\delta_t$ belong to 2-dimensional linear subspaces of $R^N$ and the results for the case $N = 2$ may be used. $\square$

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# Parameter Drift Instability in Disturbance-Free Adaptive Systems

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***Abstract*—Adaptive identification and control algorithms can exhibit local instability when certain ideal assumptions, such as satisfaction of SPR conditions, are violated. However, recent conjectures suggest that due to a "self-stabilization" mechanism, global boundedness may still hold despite local instabilities. We present counterexamples to these conjectures, showing that self stabilization is bypassed via "hidden" unbounded parameter drift. Although parameter drift instability is known to occur in adaptive systems with disturbances, concrete examples are given to show that unbounded drift can also occur in the disturbance-free case when SPR conditions are violated.**

## I. Introduction

Under ideal conditions, adaptive systems have good asymptotic properties, e.g., Lyapunov stability [1], [2], hyperstability [3], [4], and uniform exponential stability [5], [6]. This implies good system performance in the sense that parameter estimates are bounded and prediction errors converge to zero in the absence of disturbances. These ideal conditions constitute two classes of assumptions.

In the first class there are "structural" assumptions, in which the form of the unknown plant (the system to be identified, matched, or controlled) is presumed to exactly match the form of the adjustable model. Under structural matching assumptions in a deterministic analysis, there are no unmodeled dynamics, there are no unaccounted for nonlinearities, and there are no disturbances such as measurement errors or roundoff errors. In short, it is assumed the model has the capability of exactly matching the dynamics of the unknown part of the system.

In the second class there are "algorithmic" assumptions that relate the algorithm operational environment to the internal features of the adaptive system. Resulting conditions involve designer-selected coefficients (e.g., step sizes, error filters, etc.), conditions on parts of the unknown system (e.g., strictly positive real (SPR) assumptions), and conditions on the adaptive system signals (e.g., persistent excitation, persistent power).

Nonideal situations where "structural" assumptions are violated (persistent disturbances are present) have been treated by exploiting the properties of strong (i.e., uniform exponential) internal stability. If the internal stability can be made strong enough (large enough exponential convergence rate) relative to the disturbance, overall stability can be retained [5], [6]. Unfortunately, some ideal "algorithmic" assumptions, e.g., persistent excitation, are still required. In many applications, these condi-

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tions cannot be satisfied. As shown in the infamous "counterexample to adaptive control" [7], undermodeling disturbances and improper excitation lead to explosive instability. In other cases, behavior ranges from recurrent bursts in the prediction error [8]–[10] to "random" behavior resembling chaos [11], [12].

On the other hand, recent evidence [8], [9], [13]–[17] suggests that even if ideal conditions for internal stability are absent, a form of "self stabilization" can occur where overall stability may be retained. In e.g., [14], [15], SPR conditions are shown to be unnecessary if a "persistent power" condition on the information vector is satisfied. This condition can be temporarily satisfied if signals in the information vector undergo bursting, suggesting a self-stabilizing effect [16], [17] which may result in bounded limit cycle or quasi-periodic behavior in the parameter estimates. Another heuristic argument [8], [9], [13] is that bursting can cause self-generated excitation, which restabilizes the algorithm via persistent excitation conditions, in spite of possible local instability in the neighborhood of a "tuned" parameterization. However, it is known [10], [18]–[23] that bounded disturbances can "mask" the stabilizing growth of prediction errors, allowing a slow "drift" of parameters to infinity. This drift, in turn, allows unbounded bursting behavior and can result in an unstable adaptive system. Hence, self stabilization does not always occur in the presence of disturbances.

In this note, unbounded drift is shown to occur in adaptive systems even without "masking" disturbances, provided SPR conditions are violated. This parameter drift is unobservable in the prediction error, hence, the self-stabilization mechanism is defeated. The conjectures [13], [16], [17] are therefore incorrect in general. *The implication of these results is that parameter drift instability is not due solely to system disturbances.*

As the examples given here suggest, parameter drift to $\infty$ can be quite "singular," and can be classified as a "rare" event in practical applications. Even so, very large bursting in prediction errors can be readily observed as a result of drift to moderate parameter values. A variety of algorithm modifications to prevent parameter drift have already appeared, including parameter projection, normalization, leakage, and deadzones, (see e.g., [24] and the references therein). In contrast, the results of this note contribute to a more complete understanding of the underlying causes of drift instability.

## II. Disturbance-Free Parameter Drift

The class of adaptive systems considered are given by the following error model description

$$e_{k+1} = G(q^{-1})(v_{k+1}) - w_{k+1} = \phi_k^T \tilde{\theta}_{k+1} \tag{1}$$

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k - h\phi_k v_{k+1} \tag{2}$$

where

$e_{k+1}$ is the (*a posteriori*) equation error,

$G(q^{-1})$ is a monic $n$th-order polynomial in the backward shift operator $q^{-1}$,

$v_{k+1}$ is the measured (*a posteriori*) prediction error,

$w_{k+1}$ is the disturbance,

$\phi_k$ is the information vector (regressor),

$\tilde{\theta}_k$ is the parameter error vector, and

$h$ is the step size.

This type of error model occurs in a variety of adaptive systems. Two common examples are explicit model reference adaptive control, e.g., [4], and recursive identification and adaptive filtering, see e.g., [1]. Adaptive algorithms in this class are of the *a posteriori* type, which use the latest information available in the update of the parameter estimates, and are generally considered to have good stability properties [2], [4], [15].

Equations (1) and (2) form a feedback system (the error model), whose stability properties characterize the adaptive system. See Fig. 1. Disturbances $w$ to the error model are necessary for parameter drift to occur when $1/G(q^{-1})$ is SPR [1]–[4], [18]–[20]. However, when $1/G(q^{-1})$ is not SPR, drift can occur in the absence of disturbances, as shown by the following two examples.

*Example 1:* Drift of a numerator parameter in the predictor.

Plant: $y_{k+1} = \Sigma_{i=1}^5 a_i y_{k-i+1} + \theta u_k$.
*A priori* Predictor: $\bar{y}_{k+1} = \Sigma_{i=1}^5 a_i \hat{y}_{k-i+1} + \hat{\theta}_k u_k$.
*A posteriori* Predictor: $\hat{y}_{k+1} = \Sigma_{i=1}^5 a_i \hat{y}_{k-i+1} + \hat{\theta}_{k+1} u_k$.
*A Priori* prediction error (note no disturbances): $\bar{v}_{k+1} = y_{k+1} - \bar{y}_{k+1}$.
*A Posteriori* prediction error: $v_{k+1} = y_{k+1} - \hat{y}_{k+1}$.
Parameter estimate update: $\hat{\theta}_{k+1} = \hat{\theta}_k + hu_k\bar{v}_{k+1}/(1 + hu_k^2)$.

This results in the error system of (1) and (2) with

$$\tilde{\theta}_{k+1} = \theta - \hat{\theta}_{k+1} = \tilde{\theta}_k - hu_k v_{k+1} \tag{3}$$

and

$$v_{k+1} = \frac{1}{A(q^{-1})}\{\tilde{\theta}_{k+1}u_k\} = \frac{1}{G(q^{-1})}\{e_{k+1}\} \tag{4}$$

where $A(q^{-1}) = 1 - \Sigma_{i=1}^5 a_i q^{-i}$, and $e$ and $v$ are the *a posteriori* errors. For simplicity, choose $A(q^{-1}) = (q^{-1} + p)^5$ where $p^{-1} = -\tan \pi/5$. Thus, $1/A(q^{-1})$ has magnitude $m = (p^2 + 1)^{-5/2}$ and phase $-\pi$ at the frequency $\omega = \pi/2$ rad/s. To construct a drift example, choose the input $u_k$ to cause

$$e_{k+1} = \tilde{\theta}_{k+1}u_k = \sin(k\pi/2 + \pi/4) \tag{5}$$

so that when initial conditions are properly chosen, the steady-state solution is obtained:

$$v_{k+1} = -m \sin(k\pi/2 + \pi/4). \tag{6}$$

Substituting for $u_k$ and $v_{k+1}$ in the parameter error update (3) yields

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k + \frac{hm \sin^2(k\pi/2 + \pi/4)}{\tilde{\theta}_{k+1}} = \tilde{\theta}_k + \frac{hm}{2\tilde{\theta}_{k+1}}. \tag{7}$$

The two solutions for $\tilde{\theta}_{k+1}$ given $\tilde{\theta}_k$ are found from

$$\tilde{\theta}_{k+1} = \frac{\tilde{\theta}_k}{2} \pm \frac{1}{2}\sqrt{\tilde{\theta}_k^2 + 2hm}. \tag{8}$$

Taking the positive solution at each $k$ shows that $\tilde{\theta}$ is monotone increasing if $\tilde{\theta}_o > 0$. Also, from (7) a supposed bound $B$ on $\tilde{\theta}$ would imply $\tilde{\theta}_{k+1} \geq \tilde{\theta}_k + hm/(2B)$, for all $k$, contradicting any bound $B$. Hence, $\tilde{\theta}$ is unbounded. The expression (5) for $e_{k+1}$ can then be used to find the required input sequence $u$. Note $u_k \to 0$, and thus $y_k \to 0$. The predictor output $\hat{y}_{k+1}$ converges to $\sin(k\pi/2 + \pi/4)$. $\triangle\triangle\triangle$

*Remarks:*

1) The parameter estimates are unbounded in this example, but tend to $\infty$ slowly ($\tilde{\theta}_k \approx \ln(k)$), while the prediction error is bounded. This is a "nonexplosive" form of instability that has been termed [10], [25] *parameter drift*.

2) The approach of Example 1 is only used to construct the drift example, and does not correspond to an implementation of

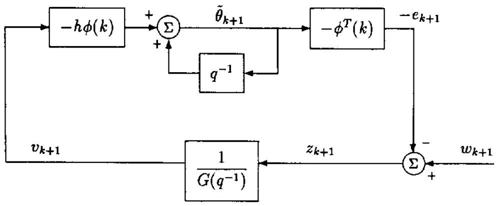


Fig. 1. Error model of the class of adaptive algorithms considered.

the adaptive system. For example, a simulation of this example may be obtained by implementing the plant, *a priori* predictor, and the parameter estimate update recursions. Given any initial parameter error (any $\tilde{\theta}_o > 0$ in this example), compute $u_{-1}$ from (5), and choose initial conditions on the plant and predictor to cause $v_k$ to equal the steady state solution (6) for $k \leq 0$. Define a parameter $\theta^*$ by

$$\theta^*_{k+1} = \frac{\theta^*_k}{2} + \frac{1}{2}\sqrt{(\theta^*_k)^2 + 2hm}\,, \qquad \theta^*_o = \tilde{\theta}_o. \tag{9}$$

and compute $u_k$ using the idea in (5):

$$u_k = \frac{\sin(k\pi/2 + \pi/4)}{\theta^*_{k+1}}. \tag{10}$$

The parameter estimate update then generates $\hat{\theta}_k$ such that $\tilde{\theta}_k = \theta^*_k$, and $\hat{\theta}_k$ drifts to $\infty$.

3) Note that the construction of the excitation sequence $u_k$ is completely "open loop." It is not computed based on a feedback of the current parameter estimate $\hat{\theta}_k$, and the entire sequence can be determined off-line, before the adaptive system is implemented.

4) The operator $1/G(q^{-1})$ in this example is not SPR, and in fact acts as a simple gain $-m$ under the sinusoidal error signals of this example. The error system is unstable in the sense that a bounded excitation signal $u_k$ exists which drives the parameter estimates to $\infty$, but the growth of $\tilde{\theta}_k$ observed in the error (5) is simultaneously "masked" by the decrease in $u_k$, and the prediction error remains bounded.

5) The prediction error can also become unbounded in systems where parameter drift occurs. As in [10], a scenario can be constructed where intervals of parameter drift are alternated with intervals of persistent excitation. Bursts in the prediction error occur when large parameter errors are suddenly "observed" on the persistent excitation intervals. Forcing parameters to drift to increasingly large values on the drift intervals produces increasingly large bursts in the subsequent excitation intervals, and both $v_k$ and $\hat{\theta}_k$ are unbounded.

In Example 1, the poles of the predictor were fixed at stable locations. The conjectures [13], [16], [17] were based on adaptive systems where predictor poles drift outside the unit circle. The instability in the predictor was seen to cause additional "self-generated" excitation, resulting in improved "observation" of parameter errors (self stabilization). However, it is also possible to force a pole of the predictor to drift to $\infty$ unobserved. Hence, self stabilization does not always occur.

*Example 2:* Drift of a denominator parameter in the predictor.

Plant: $y_{k+1} = \sum_{i=2}^5 a_i y_{k-i+1} + \theta y_k + bu_k$.
*A priori* Predictor: $\bar{y}_{k+1} = \sum_{i=2}^5 a_i \hat{y}_{k-i+1} + \hat{\theta}_k \hat{y}_k + bu_k$.
*A posteriori* Predictor: $\hat{y}_{k+1} = \sum_{i=2}^5 a_i \hat{y}_{k-i+1} + \hat{\theta}_{k+1}\hat{y}_k + bu_k$.
*A Priori* prediction error (no disturbances): $\bar{v}_{k+1} = y_{k+1} - \bar{y}_{k+1}$.
*A Posteriori* prediction error: $v_{k+1} = y_{k+1} - \hat{y}_{k+1}$.
Parameter estimate update: $\hat{\theta}_{k+1} = \hat{\theta}_k + h\hat{y}_k\bar{v}_{k+1}/(1 + h\hat{y}_k^2)$.

This results in the error system of (1) and (2) with

$$\tilde{\theta}_{k+1} = \theta - \hat{\theta}_{k+1} = \tilde{\theta}_k - h\hat{y}_k v_{k+1} \tag{11}$$

and

$$v_{k+1} = \frac{1}{A(q^{-1})}\{\tilde{\theta}_{k+1}\hat{y}_k\} = \frac{1}{G(q^{-1})}\{e_{k+1}\} \tag{12}$$

where $A(q^{-1}) = 1 - \theta q^{-1} - \sum_{i=2}^5 a_i q^{-i}$, and $e$ and $v$ are the *a posteriori* errors. Choose $A(q^{-1})$ as in Example 1 and the predictor output $\hat{y}_k$ to cause

$$e_{k+1} = \tilde{\theta}_{k+1}\hat{y}_k = \sin(k\pi/2 + \pi/4) \tag{13}$$

so that when initial conditions are properly chosen, the steady-state solution is obtained:

$$v_{k+1} = -m\sin(k\pi/2 + \pi/4). \tag{14}$$

Substituting for $\hat{y}_k$ and $v_{k+1}$ in the parameter error update yields

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k + \frac{hm}{2\tilde{\theta}_{k+1}}. \tag{15}$$

As argued for (8), the solution of (15) for $\tilde{\theta}$ is monotone increasing and is unbounded if $\tilde{\theta}_o > 0$. From (13), this implies that $\hat{y}_k \to 0$. The plant output can be found from $y_{k+1} = v_{k+1} + \hat{y}_{k+1}$ which converges to $-m\sin(k\pi/2 + \pi/4)$. The required input is given by $bu_k = A(q^{-1})y_{k+1}$ which converges to $\sin(k\pi/2 + \pi/4)$. As before, specifying initial conditions and the $u$ sequence yields unique solutions for the system signals, and $\tilde{\theta}$ drifts to infinity. △ △ △

*Remarks:*

6) An implementation of this example proceeds along the lines of Remark 1, where a recursion on $\theta^*$ is constructed based on the initial parameter error $\tilde{\theta}_o$ as in (9). The required excitation $u_k$ is then computed off-line, without feedback from the current parameter estimates $\hat{\theta}_k$ as follows:

$$\hat{y}^*_k = \frac{\sin(k\pi/2 + \pi/4)}{\theta^*_{k+1}} \tag{16}$$

$$y^*_{k+1} = -m\sin(k\pi/2 + \pi/4) + \hat{y}^*_{k+1} \tag{17}$$

$$u_k = \frac{1}{b}\left[y^*_{k+1} - \sum_{i=2}^{5} a_i y^*_{k-i+1} - \theta y^*_k\right]. \tag{18}$$

Using this bounded $u_k$, $\tilde{\theta}_k = \theta^*_k$ for all $k$, and $\hat{\theta}_k$ drifts to $\infty$.

7) Example 2 is a direct counterexample to the conjecture of [16], [17]. However, while Example 2 has the same drift behavior as Example 1, large magnitudes of $\tilde{\theta}$ are unlikely for Example 2 in simulation or in practice since the predictor itself becomes unstable. The predictor output $\hat{y}$ remains small (or goes to zero) only if the errors in computing the particular input $u_k$ are vanishingly small. Thus, the analysis holds for infinite precision calculations only, explaining why unbounded behavior was not observed in the simulation studies [8], [9], [13], [16], [17]. In this sense, an input $u_k$ which causes parameter drift can be thought of as an "open-loop control" for an unstable system [20]. As long as $u_k$ is computed accurately, the instability of the predictor is not apparent in the prediction error, and the parameter estimates continue to drift to infinity.

## IV. Conclusion

Drift instability can exist when inadequate levels of persistent excitation are present. Disturbances are necessary to cause drift when the internal SPR condition is satisfied. This paper has demonstrated that drift can also occur without disturbances when the SPR and persistent excitation conditions are violated. This unbounded parameter drift can lead to unbounded bursts in the prediction error, and self stabilization [8], [13], [16], [17] does not occur in general.

In Example 1, the drift of $\tilde{\theta}$ does not lead to an unstable predictor, in the sense that freezing $\hat{\theta}$ at some large value does not result in an unstable transfer function for the predictor. Hence, some small errors in computing the $u$ sequence which causes drift can be tolerated. This "robustness" of the drift behavior has been verified by simulation, where errors in simulation initial conditions result in the same long term drift behavior, in spite of a period of transient errors in the calculated $u$ sequence.

However, when drift occurs such that a pole of the predictor moves outside the unit circle as in Example 2, errors in computing $u_k$ are not tolerated because they excite the unstable modes of this predictor. Here, unbounded drift should not be expected to occur in practical applications, because computation and measurement errors are always available to perturb the delicate "masking" effect of the special excitation sequences causing/allowing drift. Unfortunately, even moderate amounts of parameter drift can cause large bursts in prediction errors and poor adaptive system performance.

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# Eigenstructure Assignment by Decentralized Feedback Control

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***Abstract*—The problem of eigenstructure assignment (eigenvalue and eigenvector assignment) plays an important role in control theory and applications. In this note, we introduce a new approach to eigenstructure assignment using decentralized control. First, several analytical results are presented to characterize the set of decentralized controllers which achieve desired eigenvalue assignment. Then, a method is proposed to simultaneously assign eigenvalues and eigenvectors of a linear system using decentralized control. The method is applied to the control of a power system to illustrate its effectiveness.**

## I. Introduction

The problem of eigenstructure assignment (simultaneous assignment of eigenvalues and eigenvectors) is of great importance in control theory and applications because the stability and dynamic behavior of a linear multivariable system are governed by the eigenstructure of the system. In general, the speed of the dynamic response of a linear system depends on its eigenvalues whereas the "relative shape" of the dynamic response depends on the associated eigenvectors. Eigenstructure assignment by

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