Detecting and Manipulating Musical Rhythms

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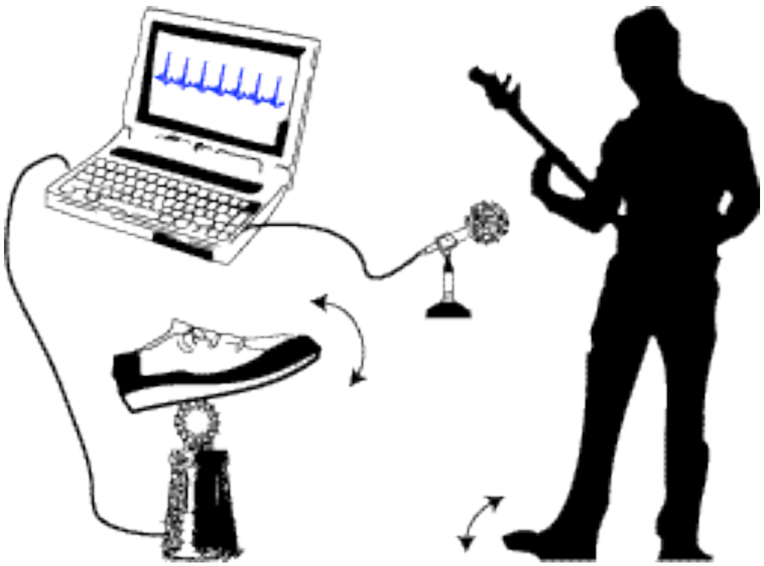
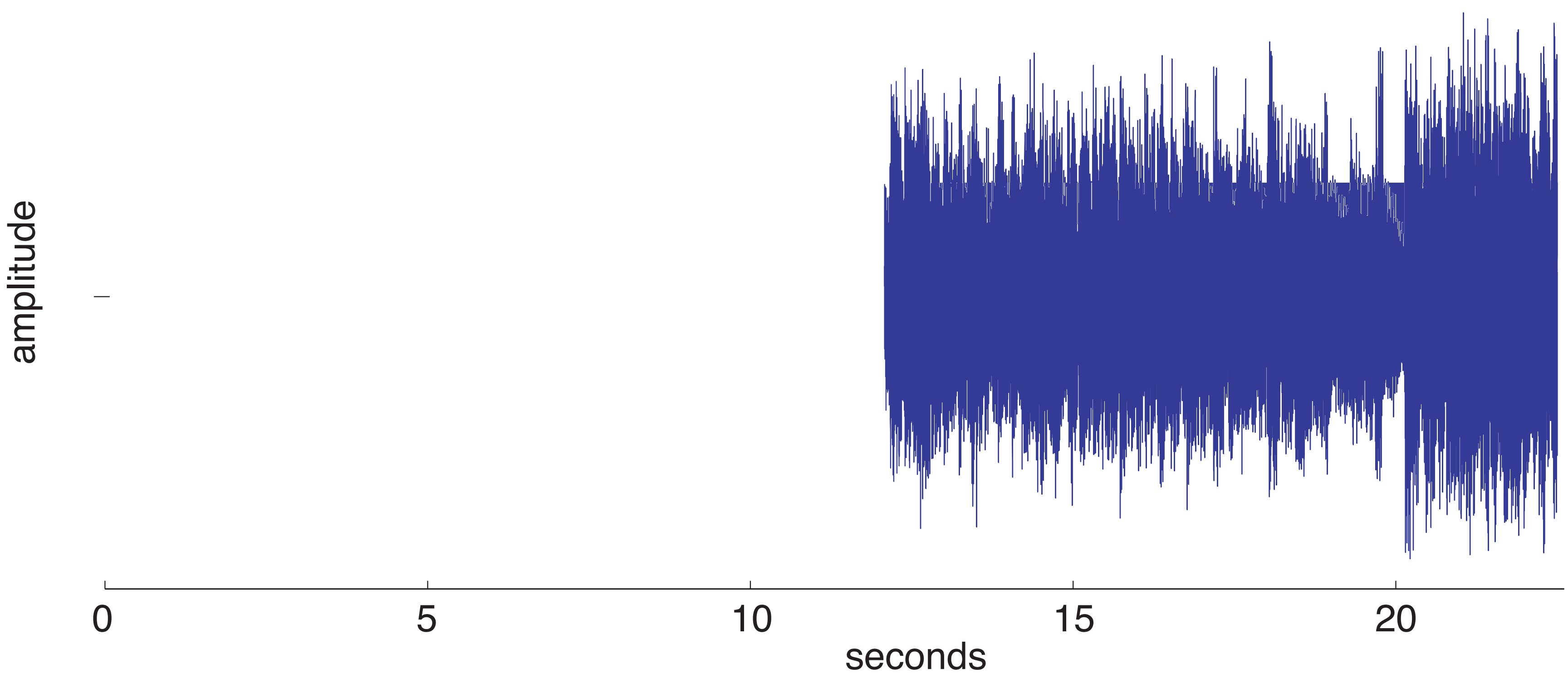
Beat tracking methods attempt to automatically synchronize to complex nonperiodic (yet repetitive) waveforms; to create an algorithm that can “tap its foot” in time with the music. Important for

- musical signal processing
- information retrieval



First 1,000,000 samples of the *James Bond Theme*

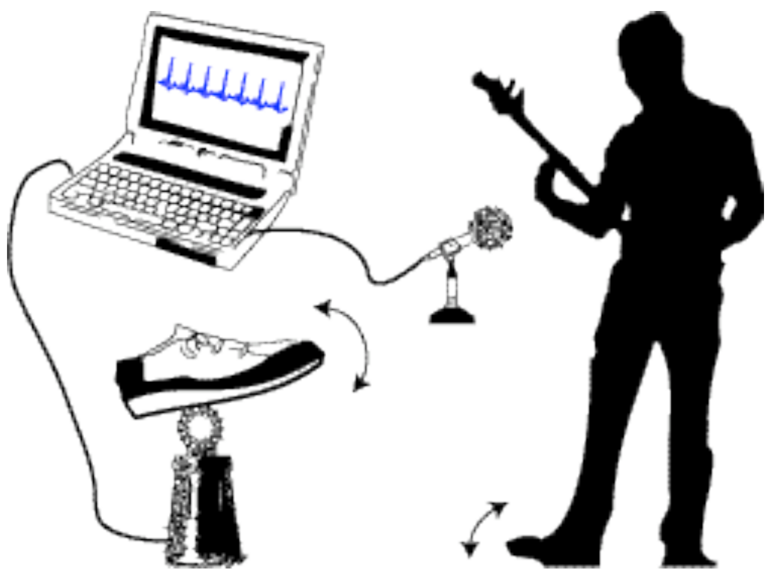


The Audio Beat Tracking Two-Step

Step 1: Parse the audio into feature vectors: may be physical parameters (energy, spectral measures, etc.) or perceptual features (note events, interonset intervals, etc.)

Step 2: Locate patterns using:

- Transforms (Fourier, Wavelet, filter banks, autocorrelations, periodicities, etc.)
- Statistical Techniques (Kalman filters, Bayesian analysis, etc.)
- Dynamical Systems (oscillators)



Two Ideas:

The first is a method of data reduction that creates a collection of *rhythm tracks* (feature vectors) which represent the rhythmic structure of the piece. Each track uses a different method of (pre)processing the audio, and so provides a (somewhat) independent representation of the beat.

The second idea is to model the rhythm tracks (in simplified form) as a collection of random variables with changing variances: the variance is small when "between" the beats and large when "on" the beat. Exploiting this simple stochastic model of the rhythm tracks allows the beat detection to proceed using Bayesian methods.

Applications of Beat Tracking

- helping to understand how people process temporal information
- editing of audio data
- synchronization of visuals with audio
- audio information retrieval
- audio segmentation and signal processing
- a drum machine that "plays along with the band" rather than the band playing to the machine

What is a "beat" anyway?

Definition 1 *An auditory boundary occurs at a time t when the sound stimulus in the interval $[t - \epsilon, t]$ is perceptibly different from the sound stimulus in the interval $[t, t + \epsilon]$.*

Definition 2 *A beat is a regular succession of auditory boundaries.*

For example, a series of audio boundaries occurring at times

$$\tau,\ \tau + T,\ \tau + 2T,\ \tau + 3T,\ \tau + 4T, \ldots$$

forms a beat of tempo T with a "phase" of τ.

Idea of the rhythm track model

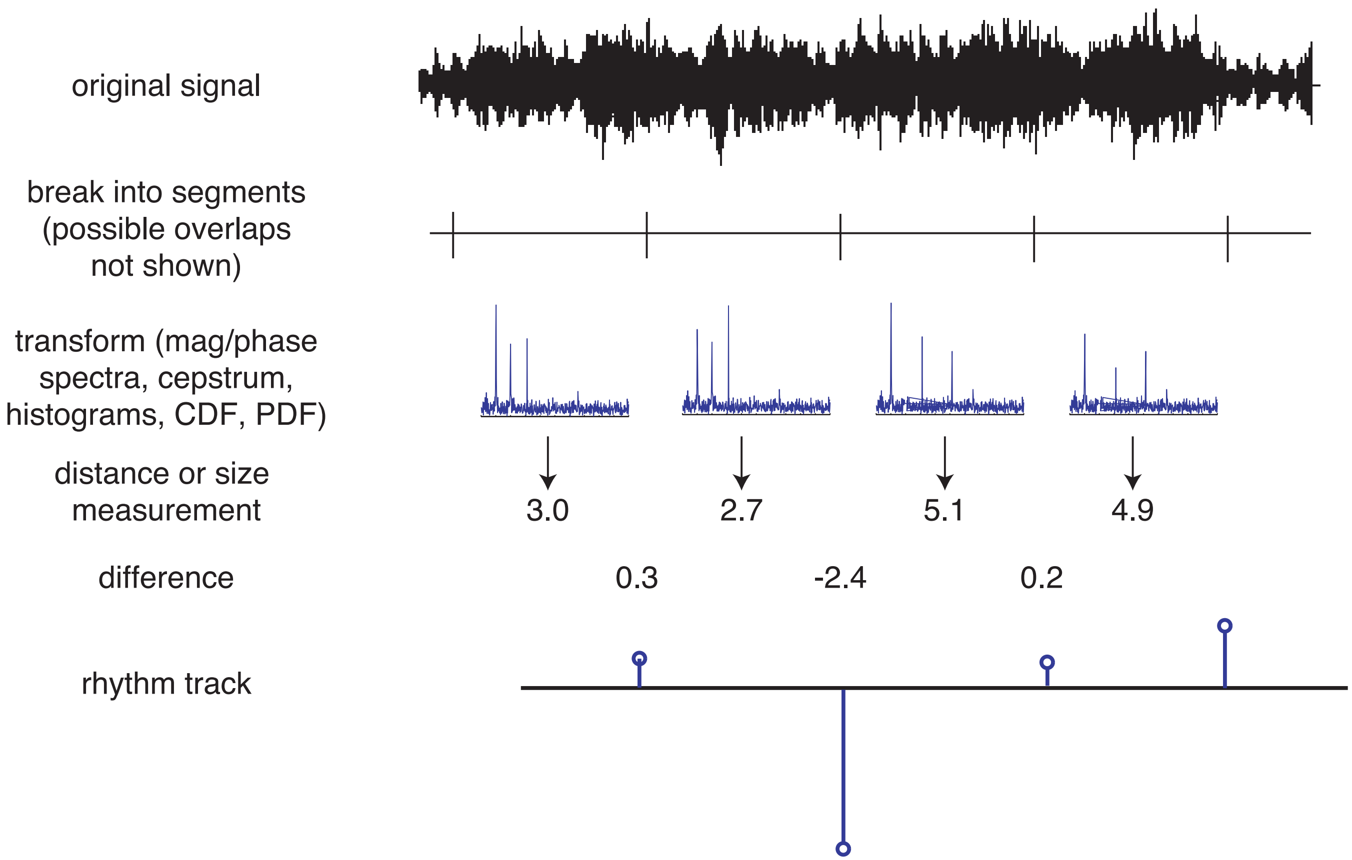
- T is on the order of 100ms-2s (hence want data reduction by a factor of ≈ 100).

- Beats tend to occur at auditory boundaries, when the sound changes.

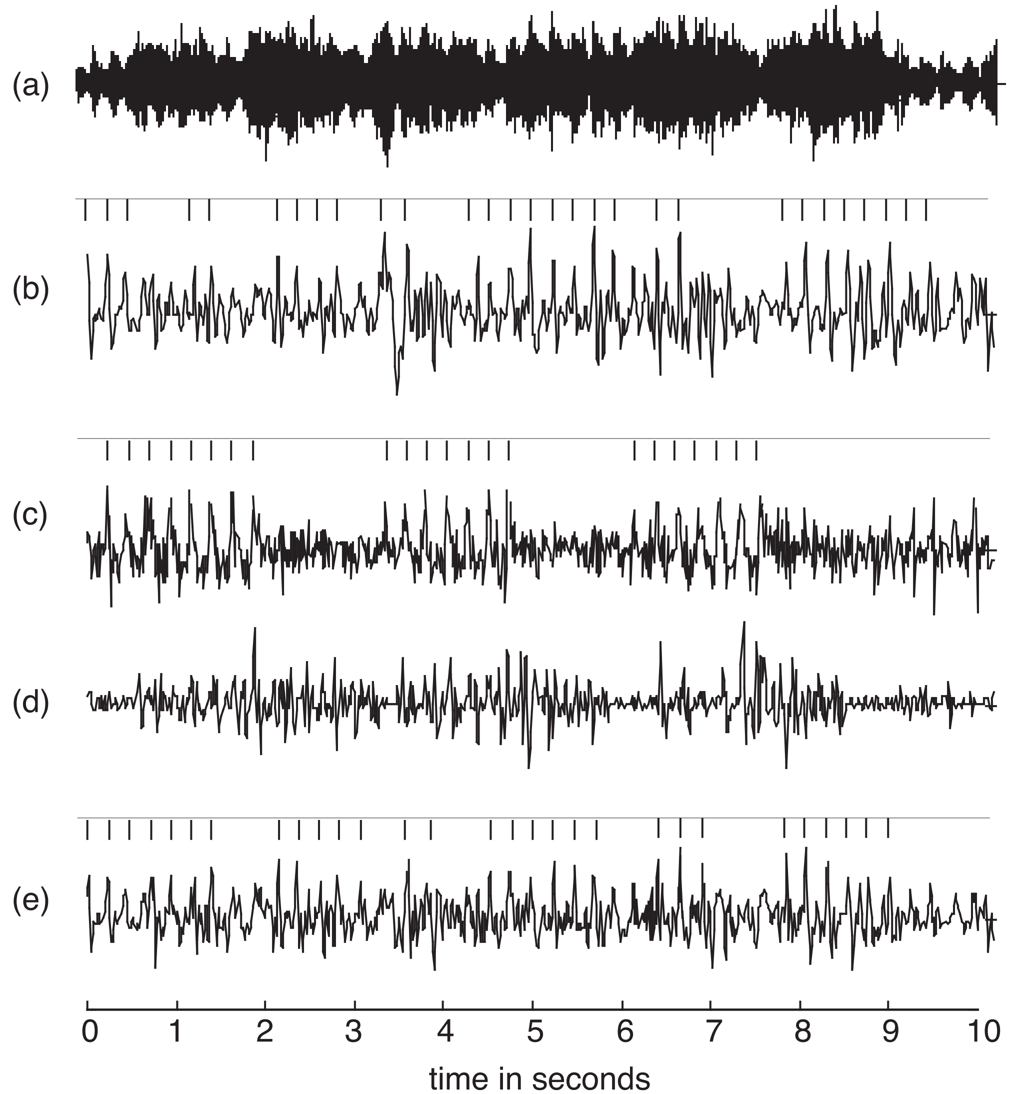
- Many things may cause boundaries (amplitude changes, pitch/frequency changes, changes in timbre/spectrum, etc.) Will measure (initially) four different aspects.

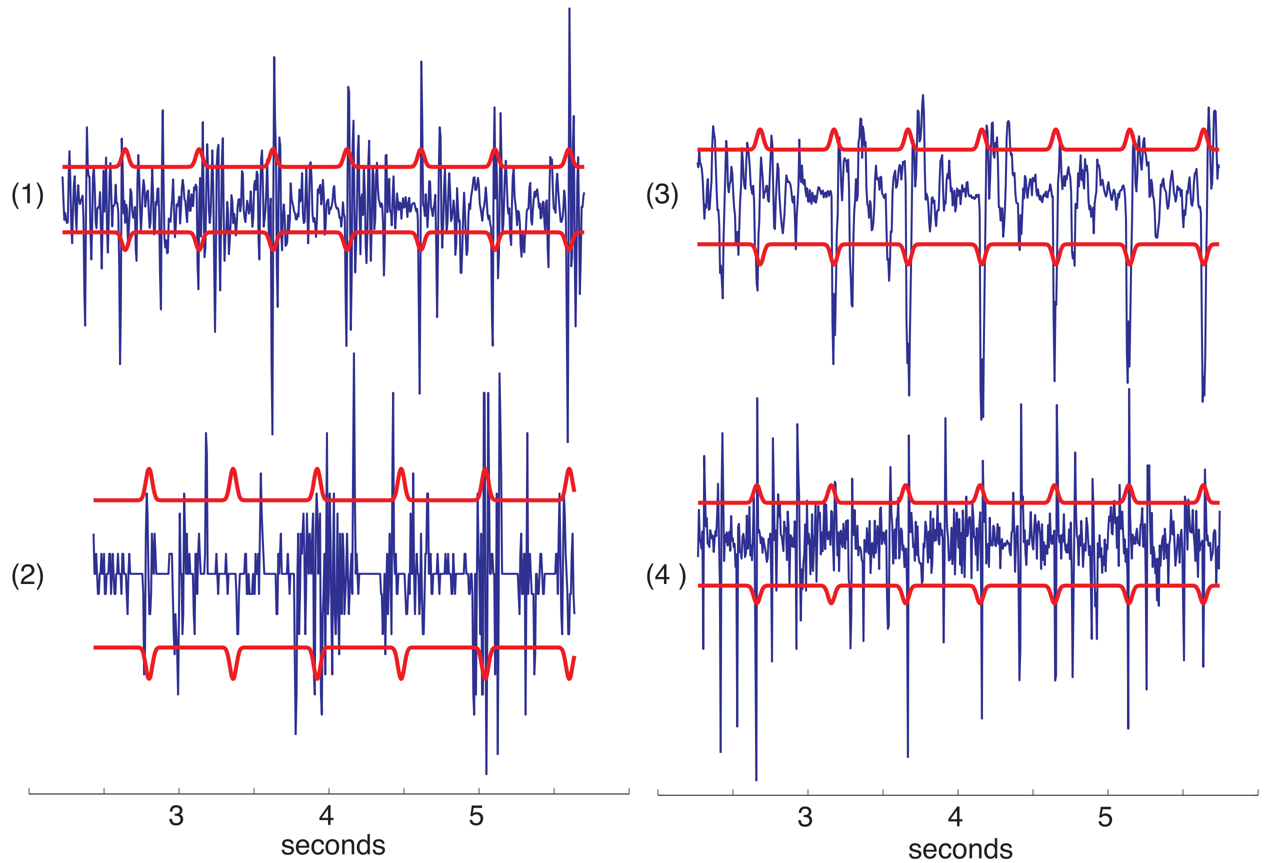
- Because they look at different characteristics of the audio (each of which is related to the rhythmic aspects), they may be considered quasi-independent observations. Need way to combine these.

Building a rhythm track feature vector



Four rhythm tracks applied to the first 10 seconds of a recording of Handel's *Water Music*: part (a) the audio waveform, (b) the energy method, (c) group delay, (d) center of the spectrum, and (e) the dispersion. Tick marks emphasize beat locations that are visually prominent.





The four rhythm tracks of *Pieces of Africa* by the Kronos quartet between 2 and 6 seconds. The estimated beat times (which correctly locate the beat in cases (1), (3), and (4)) are superimposed over each track.

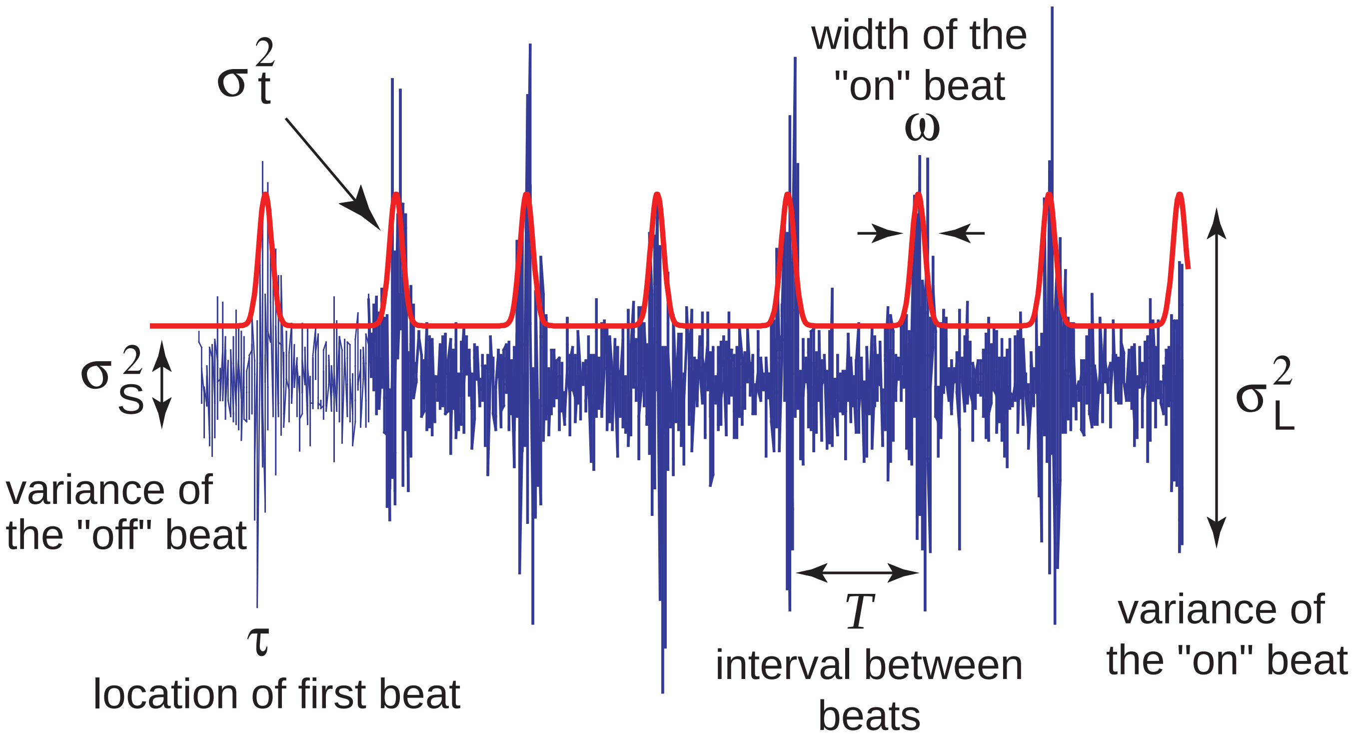
Parameters of the rhythm track model

Structural parameters:

- σ_1^2 is the "off the beat" variance,
- σ_2^2 is the "on the beat" variance, and
- ω is the beatwidth, the variance of the width of each set of "on the beat" events. For simplicity, this is assumed to have Gaussian shape.

Timing parameters:

- τ is the time of the first beat
- $\mathcal{T}$ is the period of the beat, and
- $\delta\mathcal{T}$ is the rate of change of the beat period.

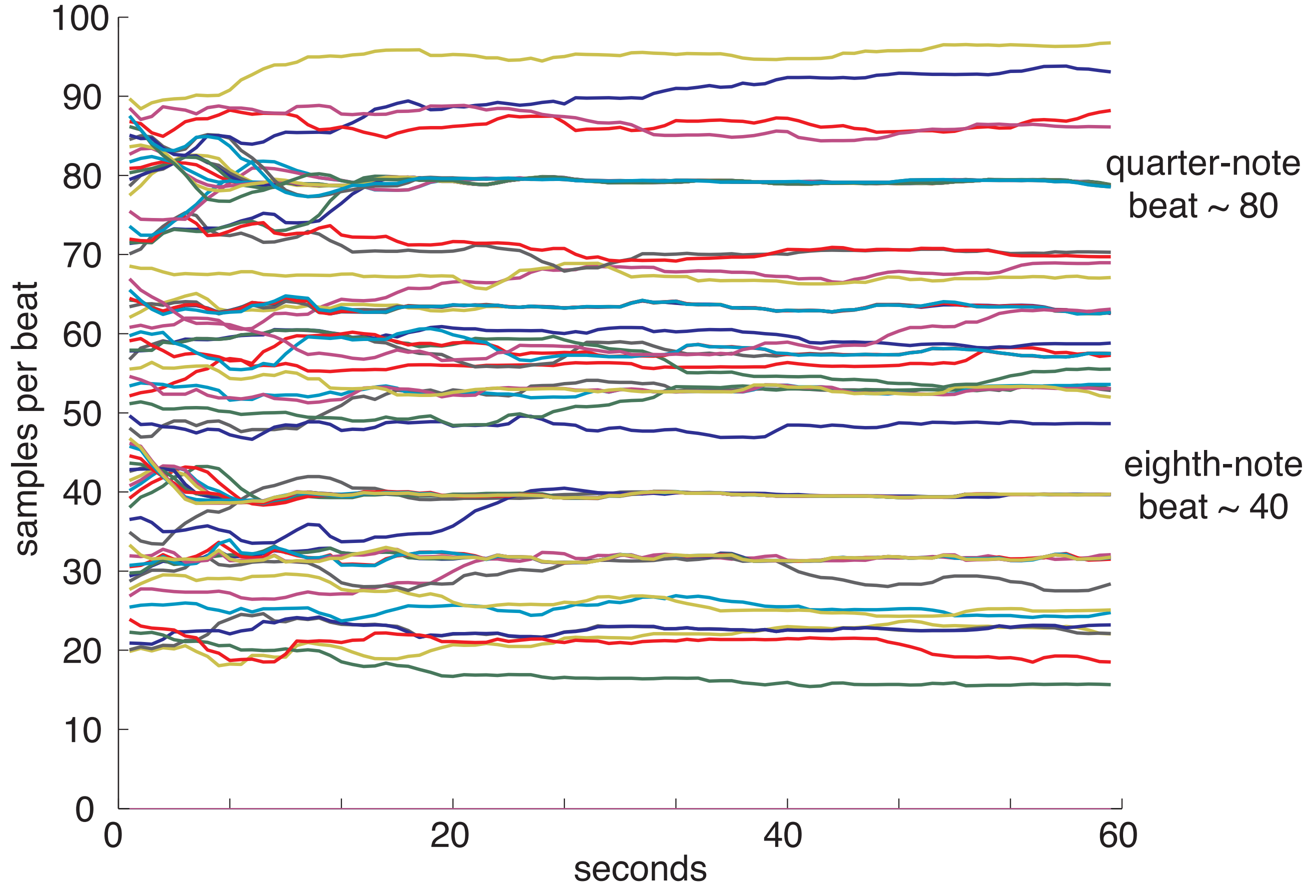


Parameters of the rhythm track model are $\mathcal{T}$, τ, ω, σ_1, σ_2 and $\delta\mathcal{T}$ (not shown). Generative model assumes rhythm tracks composed of normal zero-mean random variables with variances defined by σ's.

So now we have ways of transforming the raw audio into rhythm tracks. Second idea is to somehow parse the rhythm tracks in order to identify the parameters. Three possibilities:

- Adaptive oscillator (gradient techniques – like a PLL for sound)
- Bayesian approaches
- Transform techniques (correlations with various basis functions)

Gradient-specified oscillators are straightforward to implement, and computationally fast. But. . .



Estimates of the beat period for the *Theme from James Bond* using a gradient algorithm. Depending on initialization it may converge to a 1/8-note beat near 40 samples per period (0.23s) or to the 1/4-note beat near 80 (0.46s).

Bayes Rule

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

relates the conditional probability of A given B to the conditional probability of B given A.

If we think of A as data and B as a hypothesis, then Bayes rule says that

$$P(\text{Hypothesis}|\text{Data}) \sim P(\text{Data}|\text{Hypothesis})$$

This is useful because we want to know the left hand side (which we can't calculate) while it is often possible to calculate the right hand side numerically. In our application,

$$P(\text{Timing Parameters}|\text{Feature Vector}) \sim P(\text{Feature Vector}|\text{Timing Parameters})$$

Applying a Bayesian Model

Collect the timing parameters, $\tau, \mathcal{T}$ and $\delta\mathcal{T}$ into a state vector **t**, and let $p(\mathbf{t}_{k-1}|\cdot)$ be the distribution over the parameters at block $k-1$. The goal of the (recursive) particle filter is to update this to estimate the distribution over the parameters at block k, that is, to estimate $p(\mathbf{t}_k|\cdot)$.

The predictive phase details how $\mathbf{t}_k$ is related to $\mathbf{t}_{k-1}$ in the absence of new information: a diffusion model.

The update phase incorporates new information from the current time block.

Tracking Using Particle Filters

- $p(\mathbf{t}_k|$current block of rhythm tracks) is proportional to $p($current block of rhythm tracks$|\mathbf{t}_k)p(\mathbf{t}_k|$previous block of rhythm tracks)

- Because the rhythm tracks are considered to be independent, the posterior is

$$\Pi_i \; p(\text{rhythm track } i|\mathbf{t}_k)p(\mathbf{t}_k|\text{previous block of rhythm tracks})$$

- $p($rhythm track$|\mathbf{t}_k)$ is modeled as a product of Gaussians with the structured pattern of variances given above.

Particle Filters II

Applied to the beat tracking problem, the particle filter algorithm can be written in three steps. The particles are a set of N random samples, $\mathbf{t}_k(i), i = 1 \dots N$ distributed as $p(\mathbf{t}_{k-1}|\mathbf{R}_{k-1})$.

1. **Prediction:** Each sample is passed through the system model to obtain samples of

$$\mathbf{t}_k^{\dagger}(i) = \mathbf{t}_{k-1}(i) + w_{k-1}(i) \;\text{ for }\; i = 1, 2, \dots, N,$$

which adds noise to each sample and simulates the diffusion portion of the procedure, where $w_{k-1}(i)$ is assumed to be a 3-dimensional Gaussian random variable with independent components. The variances of the three components depend on how much less certain the distribution becomes over the block.

2. **Update:** with the new block of rhythm track values, r_k, evaluate the likelihood for each particle. Compute the normalized weights for the samples

$$q_i = \frac{p(r_k|\mathbf{t}_k^\dagger(i))}{\sum_i p(r_k|\mathbf{t}_k^\dagger(i))}.$$

3. **Resample:** Resample N times from the discrete distribution over the $\mathbf{t}_k^\dagger(i)$'s defined by the q_i's to give samples distributed as $p(\mathbf{t}_k|\mathbf{R}_k)$.

How well does it work?

OK.

Listen to a couple of examples. What we've done is to superimpose a short noise burst at each beat – hence it's easy to hear when things are working and when they're not.

- *Theme from James Bond* (bondtap)
- Handel's *Water Music* (watertap)
- Brubeck's *Take Five* (take5tap)
- Joplin's *Maple Leaf Rag* (maplecoolrobtap)
- Baltimore Consort's *Howell* (BChowelltap)

Examples of some things you can do when you know the beat structure of a piece of music:

- Manipulate on a per-beat basis (e.g., *Reversed Rag*) (maplecoolreverse)
- An ideal situation for FFT-based analysis (e.g., *Rag Minus Noise*, *Rag Minus Signal*) (maplecoolnoise) (maplecoolsig)
- Edit/add stuff (e.g., *Switched on Rag*) (maplecoolrobdrums)
- Re-order a piece 1-2-3- . . . - 30-31-32-31-30-29 - . . . -3-2-1 (friendneirf)
- Manipulate structure of piece (e.g., the *Maple Leaf Waltz*, *James Bond Waltz*, *Backwards Bond*, *Take 4*) (maple34) (bond34) (bondback) (take4)

Beat-Based Signal Processing. . .

- David Bowie singing backwards. . . (New Star City)
- Map Maple to 5-tet (Pentatonic Rag)
- Map Maple to many n-tets (maplemanytet)

A

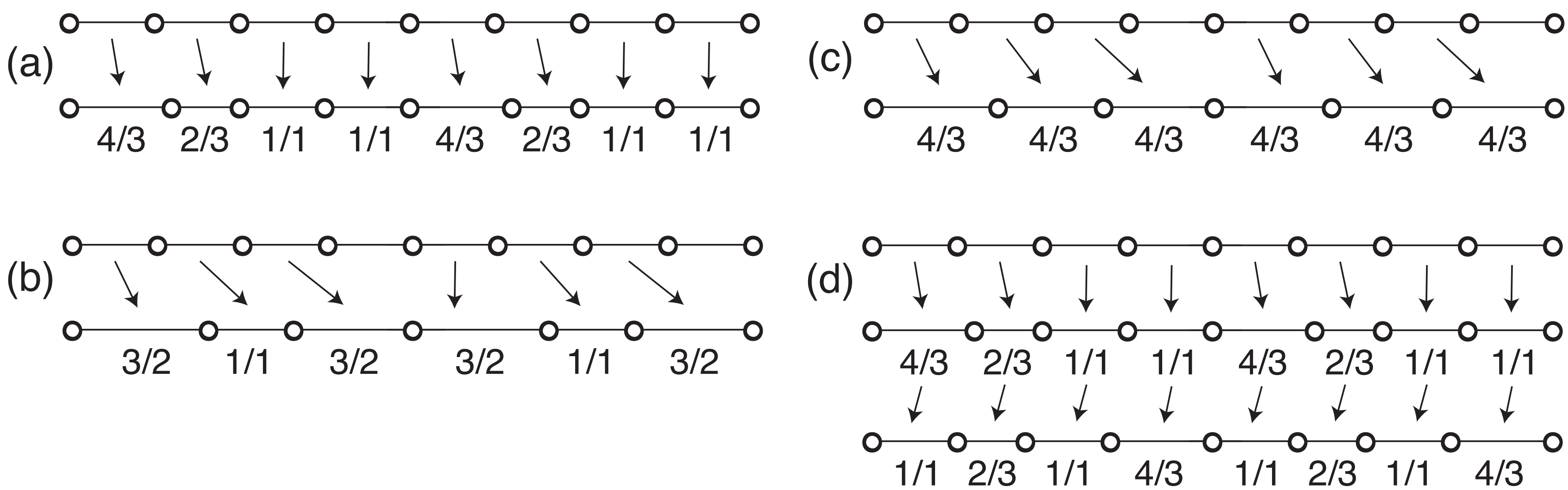
n-tet destination	3	3	3	4	4	4	4	5	5	5	5	7	7	7	7	3
beat	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

B

n-tet destination	4	4	4	5	4	4	4	5	5	5	4	5	5	5	4	4
beat	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Beat-Based Signal Processing II. . .

Changing the duration of beat intervals can be used as a kind of beat-synchronized delay processing. Performing different versions simultaneously increases the density, often in a rhythmic way.



(MagicLeafRag, MakeItBriefRag)

Beat-Based Signal Processing III. . .

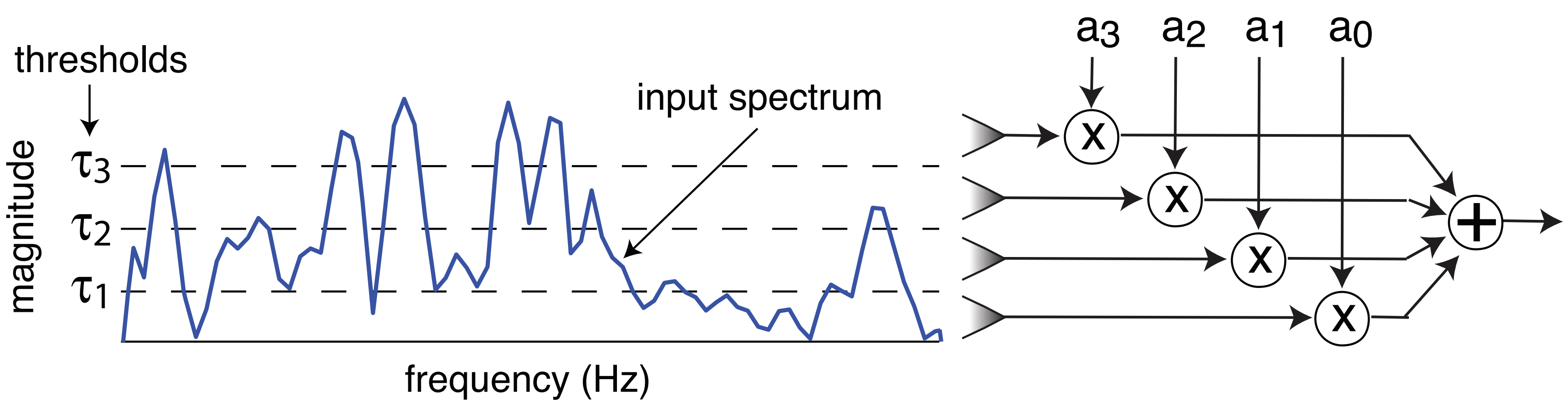
Using complex waveforms (like the *Maple Leaf Rag*) as an input to a synthesizer, carrying out synthesis on a per-beat basis.

- *Beat Gated Rag* (BeatGatedRag)

- *Noisy Souls* and *Frozen Souls* (NoisySouls, FrozenSouls)

Beat-Based Signal Processing IV. . .

Spectral band filters sound radically different from any linear filter. *Local Variations* results from application of a fixed (eight band) spectral band filter to *Local Anomaly*. Within each beat, the relative sizes of the spectral peaks are rearranged, causing drastic timbral changes that nonetheless maintain the rhythmic feel.

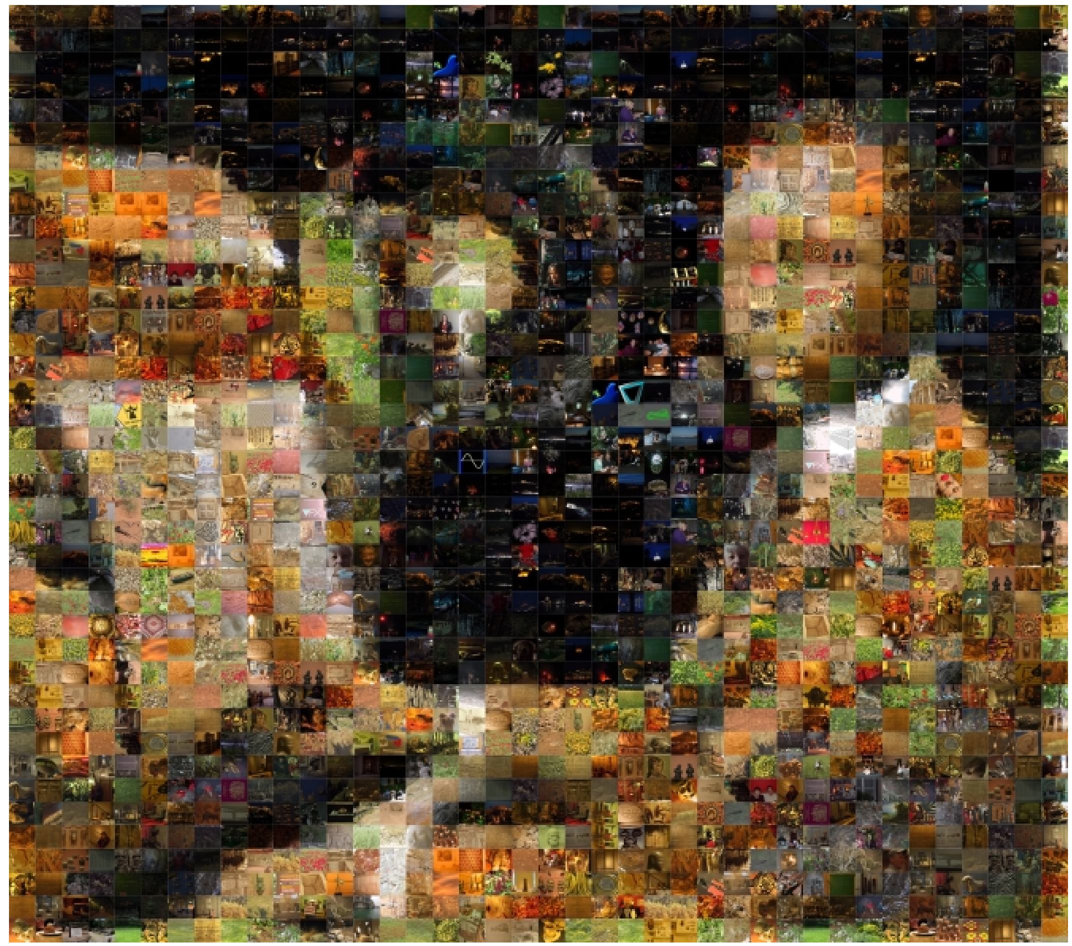


(LocalAnomaly, LocalVariation)

Beat-Based Signal Processing V. . .

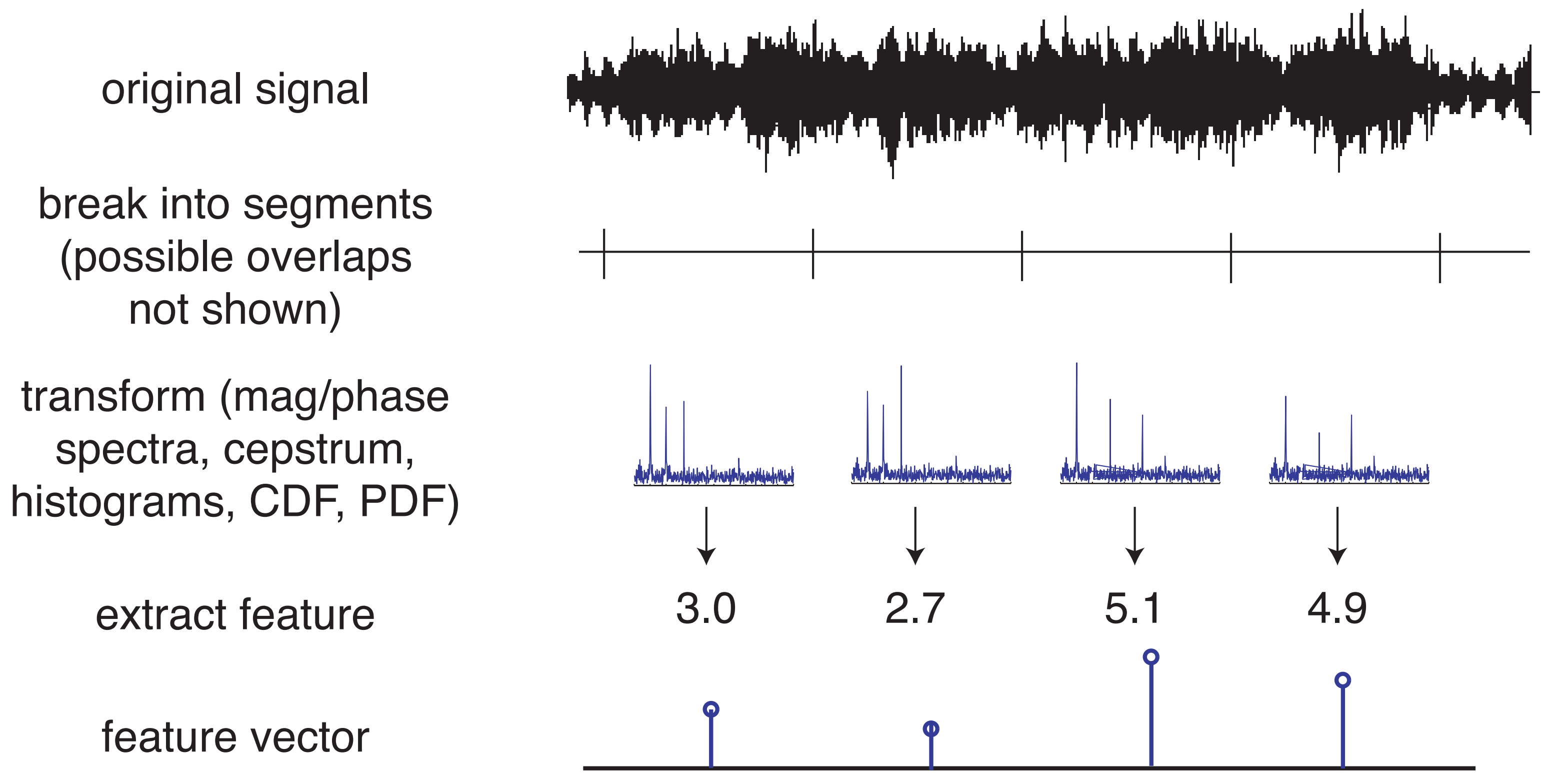
Can generate "new" pieces from old. Twenty-nine versions of Scott Joplin's *Maple Leaf Rag* (piano, marimba, vocal, orchestral, etc.) are beat tracked and aligned. A new version is created by adding together four randomly chosen versions at each beat, forming a sound collage.

(Ragbag1, Ragbag2)



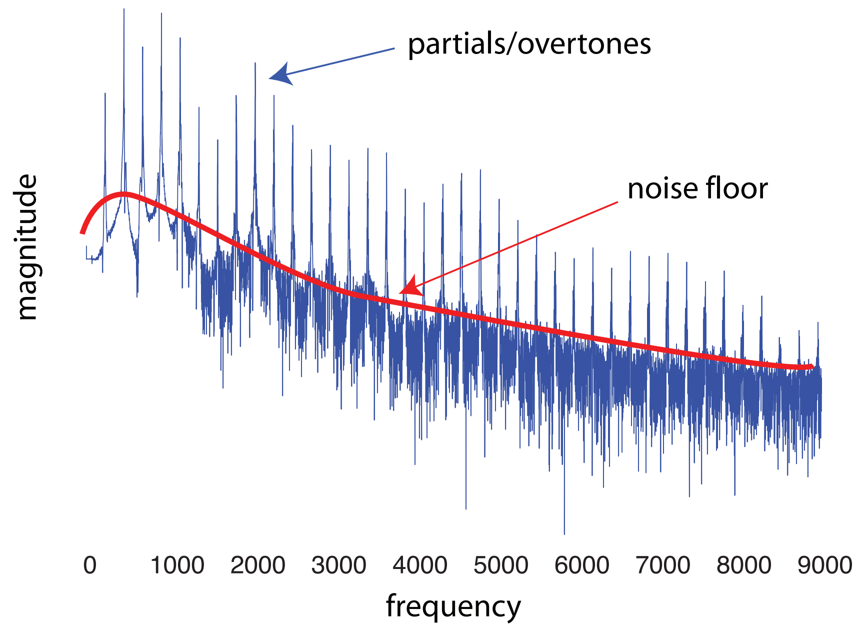
	Phase Vocoder	Beat-Synchronized FFT
windows	small frames from 1K-4K with 2 to 8 times overlap	large beat-sized windows $\frac{1}{5}$-$\frac{1}{2}$ sec, zero padded to a power of two
FFT resolution	40 Hz - 10 Hz (improved by phase adjustment)	3 Hz - 1.5 Hz (phase adjustment possible)
peak finding	all local max above median or threshold	plus distance parameter (forbidding peaks too close together)
spectral mapping	direct resynthesis: output frequencies placed in FFT vector with phase adjustment	resampling with identity window, no phase adjustment
beat detection	optional	required
examples	Maple5tetPV	Maple5tetFFT
	Soul65HzPV	Soul65HzFFT

Feature Vectors attempt to extract relevant features of the sound from the waveform by reducing it to frames and deriving a single number from each frame



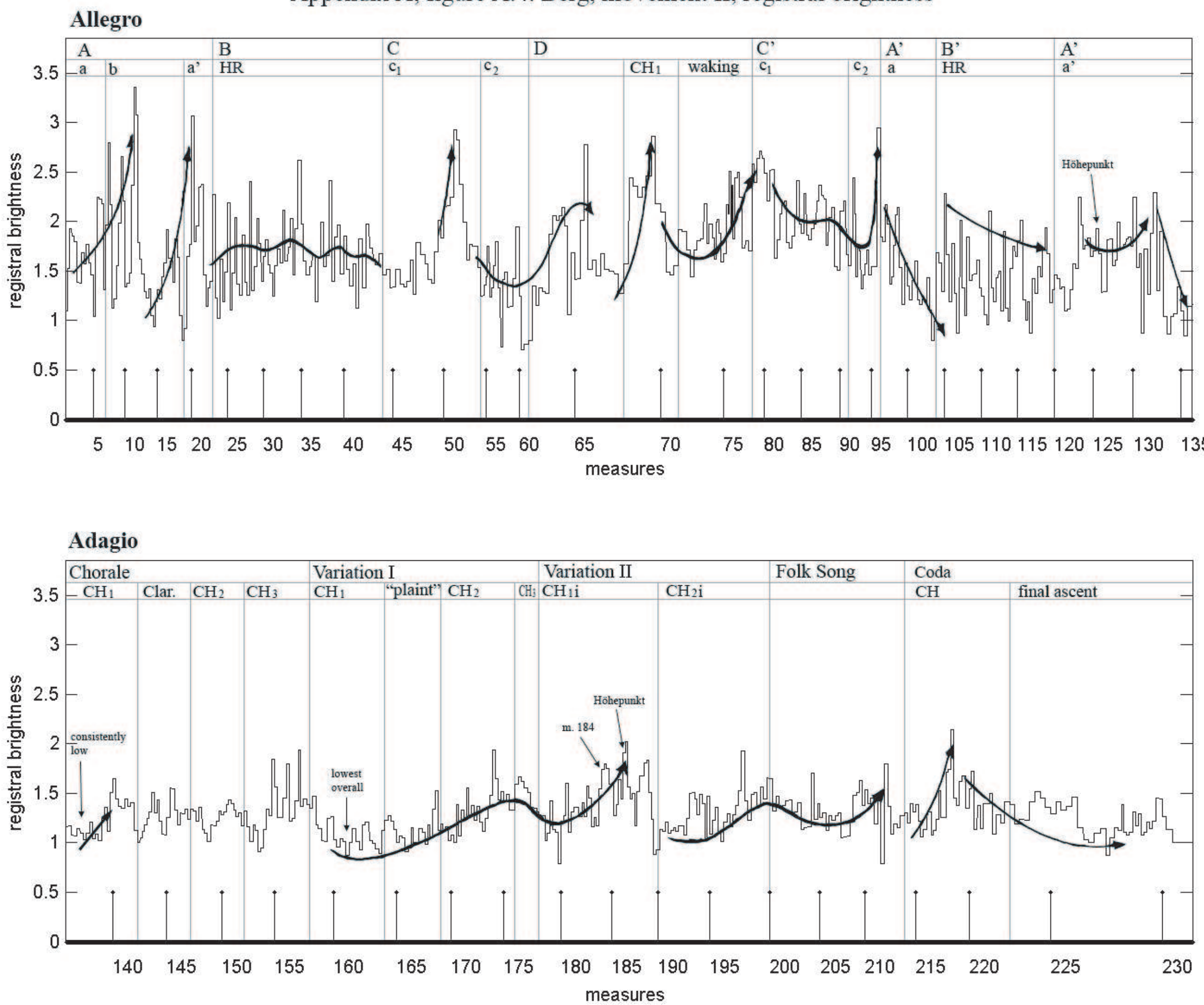
Some Interesting Features

Sensory dissonance (within each frame)
Centroid of magnitude spectrum
Dispersion about centroid
Signal to Noise ratio
Number of significant partials
Slope of Noise Floor

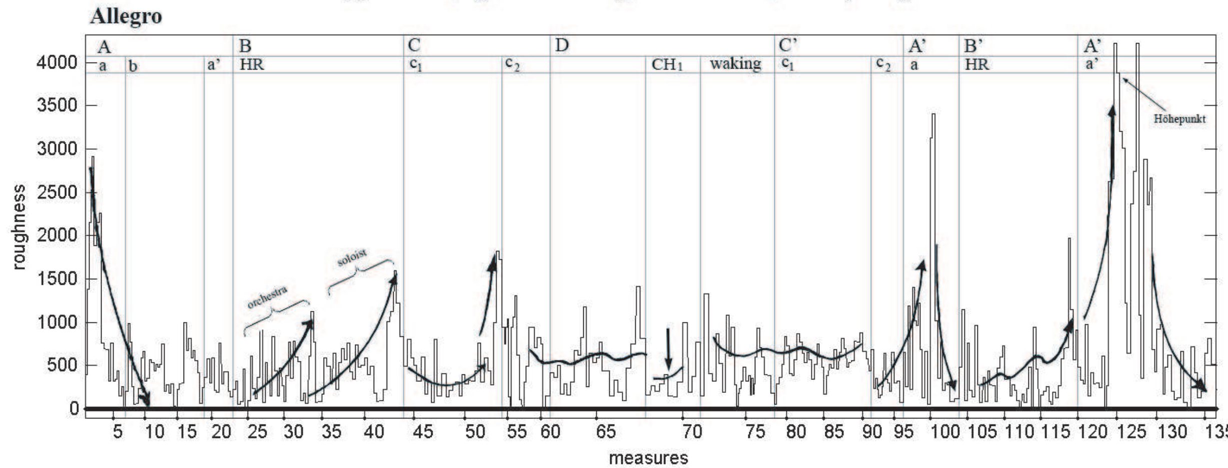


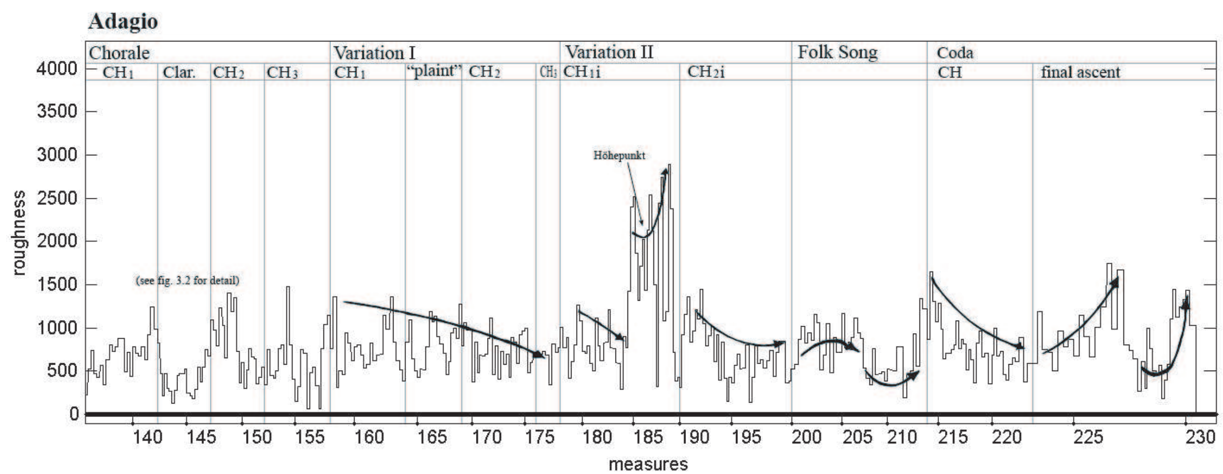
Some examples using Berg's Angel Concerto. . .

Appendix A, figure A.4. Berg, movement II, registral brightness

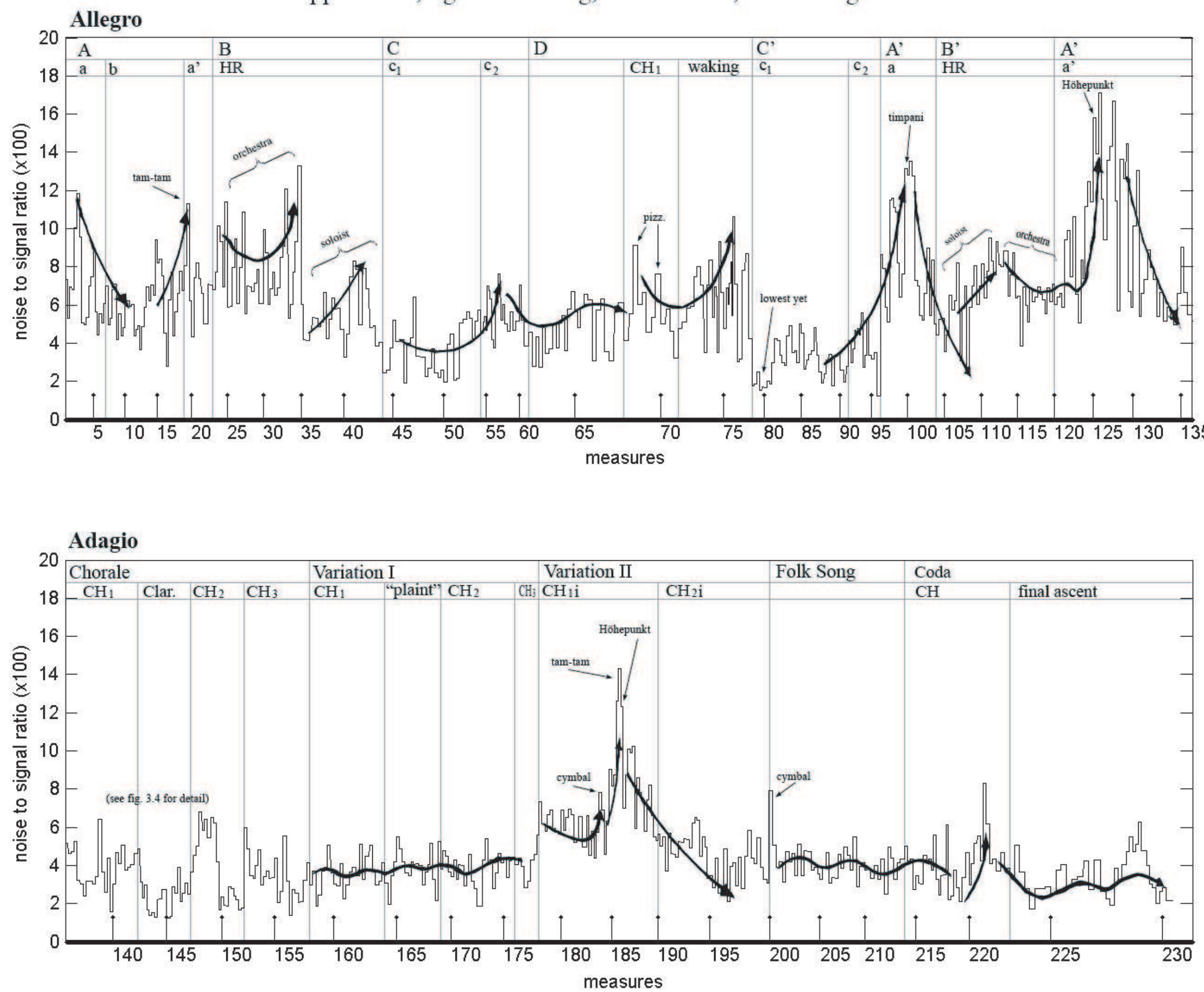


Appendix A, figure A.1. Berg, movement II, sensory roughness





Appendix A, figure A.3. Berg, movement II, noise to signal ratio



Of course, method can't possibly work when music is inappropriate, e.g., swirling undifferentiated sound masses with no discernable rhythm. But, when the music is appropriate, it still does not always work.

What are the failure modes?

When the algorithm cannot find the correct beat times, is it that

- the rhythm tracks fail to correctly locate beat boundaries?
- the Bayesian algorithm fails to find τ, T, and/or dT despite good rhythm tracks?

What other kinds of rhythm tracks can we think of?

	other ways to measure distance	
1	$\sum_i x_i^2$	l^2 energy
2	$\sum_i \lvert x_i \rvert$	l^1 norm
3	$\sum_i \log(x_i^2)$	log energy
4	$\sum_i x_i^2 \log(x_i^2)$	entropy
5	$\sum_i \lvert x_i \rvert \log(\lvert x_i \rvert)$	absolute entropy
6	argmax $\lvert x_i \rvert$	location of maximum
7	max $\lvert z_i - y_i \rvert$	KS test (for CDF)
8	$\#\{i : \lvert z_i \rvert > \eta\}, \eta = \text{mean}(z_i)$	number of $\lvert z_i \rvert$ larger than mean
9	$\eta = \sqrt{2 \log_e(n \log_2(n))}$	SURE threshold in measure 8
10	$\max(z_i) - \min(z_i)$	range of data
11	$\min_m \sum_i (z_i - mi - b)^2$	slope
12	$\min_m \lvert \sum_{j=1}^{m} z_j^2 - \frac{1}{2} \sum_{i=1}^{n} z_i^2 \rvert$	center
13	$\sum_i (m - z_i^2)^2$	dispersion about center m
14	$\sum_i \lvert z_{i+1} - z_i \rvert$	total absolute variation
15	$\sum_i (z_{i+1} - z_i)^2$	total square variation
16	$\sum_i \log(\frac{\lvert z_i \rvert}{\lvert y_i \rvert})$	cross information
17	$\sum_i \lvert z_i \rvert \log(\frac{\lvert z_i \rvert}{\lvert y_i \rvert}) + \lvert y_i \rvert \log(\frac{\lvert y_i \rvert}{\lvert z_i \rvert})$	symmetrized cross entropy
18	$\sum_i i z_i^2$	weighted energy

The partitioned data can be transformed into different domains.

label	domain
A	time signal
B	magnitude of FFT
C	phase of FFT
D	cepstrum
E	PDF of time signal
F	CDF of time signal
G	FFT of the PDF of time
H	PDF of FFT magnitude
I	CDF of FFT magnitude
J	PDF of cepstrum
K	CDF of cepstrum
L	various subbands

How many different ways of building rhythm tracks are there?

Approximately the product of:

$$\left\{ \begin{array}{c}\text{\# ways to}\\ \text{choose partitions}\end{array} \right\} \times \left\{ \begin{array}{c}\text{\# of}\\ \text{domains}\end{array} \right\} \times \left\{ \begin{array}{c}\text{\# of distance}\\ \text{measures}\end{array} \right\} \times \left\{ \begin{array}{c}\text{\# ways of}\\ \text{differencing}\end{array} \right\}$$

We found 7344 different rhythm tracks. Need a way of testing to see if these are good or bad.

Idea for a test

Since rhythm tracks may be modeled as a collection of normal random variables with changing variances, can measure the quality Q of a rhythm track by measuring the fidelity of the rhythm track to the model.

(a) Choose a set of test pieces for which the beat boundaries are known.

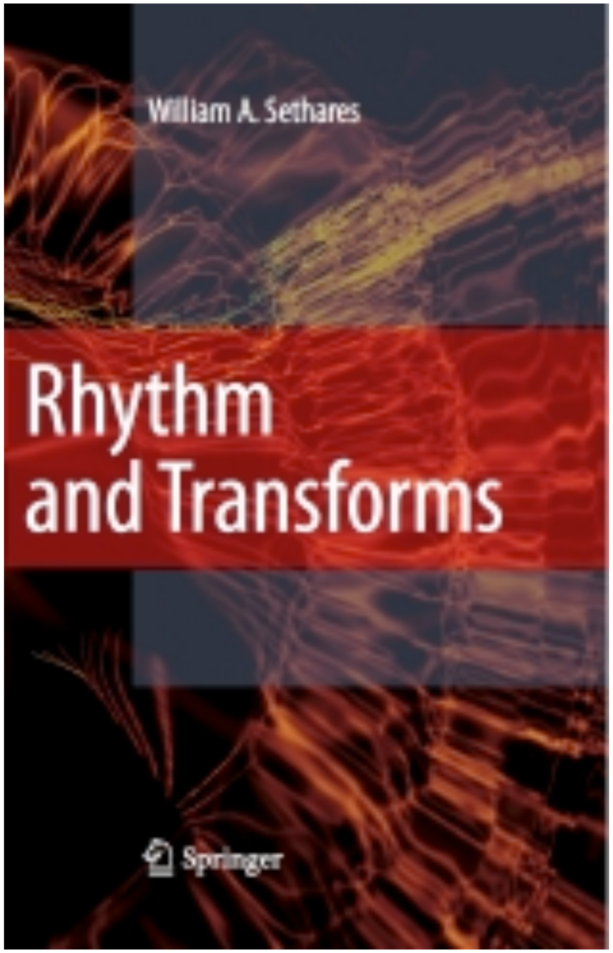
(b) For each piece and for each candidate rhythm track, calculate the quality measure Q.

(c) Those rhythm tracks which score highest over the complete set of test pieces are the best rhythm tracks.

(d) Independence: check that the rhythm tracks are truly independent of eeach other (e.g., SVD test).

The best rhythm tracks

- based on magnitude of FFT
- based on CDF/PDF (histograms) of FFT
- based on CDF/PDF (histograms) of cepstrum
- the only time-based measure remaining was energy
- none of standard stochastic tests (SURE, K-S, etc.) based on time signal, but some using CDFs



Rhythm and Transforms focuses on the technologies of beat tracking, and describes the impact of beat tracking on music theory and on the design of sound processing electronics such as musical synthesizers, drum machines, and special effects devices. Includes over five hours of sound examples on CD!